Published in Cryogenics 32 (1992) 843

Ideal Orifice Pulse Tube Performance

Peter Kittel

NASA, Ames Research Center, Moffett Field, CA 94035, U.S.A.

Using an analogy to the Stirling cycle refrigerator, the efficiency (cooling power per unit input power) of an ideal orifice pulse tube refrigerator is shown to be T_1/T_0, the ratio of the cold temperature to the hot temperature.

Keywords: pulse tubes; refrigerators; efficiency

The recent development of orifice pulse tube refrigerators (OPT)[1] has raised questions of what limits their ultimate performance. It has become recognized that these refrigerators operate on a thermodynamic cycle close to the Stirling cycle. This paper presents an analysis of the efficiency of an ideal OPT, thus placing a limit on OPT performance. The analysis is based on making an analogy between the Stirling cycle and the OPT cycle and is based on the analysis of Kasuya, et. al.[2] Both the Stirling and OPT cycles involve oscillating gas flows. For the purpose of this discussion all thermodynamic quantities are averaged over one cycle. This removes from consideration effects which average to zero. This approach is valid for ideal, lossless devices. However, in practical machines, this approach can mask losses.

The ideal Carnot cycle is a good starting point for this discussion. The energy flow for this cycle is shown in fig. 1. From the conservation of energy:

$$Q_1 + W_n = Q_0. \tag{1}$$

In an ideal, lossless, system entropy is also conserved:

$$Q_1/T_1 = Q_0/T_0. \tag{2}$$

The efficiency is then

$$\square_c \equiv Q_1/W_n = (T_0/T_1 - 1)^{-1}; \tag{3}$$

where (1) and (2) have been used. The specific power, the input power needed per unit of cooling, is

$$P_c = 1/\square_c = T_0/T_1 - 1. \tag{4}$$

A Stirling refrigerator in the □ configuration is shown schematically in fig. 2. This cooler has a warm compressor, a regenerator, two heat exchangers, and a cold expander. The motion of the two pistons is phased to produced a phase shift between the pressure oscillation and compressor motion. This produces a work flow and cooling at the expander. In an ideal system the compressor and expander operate isothermally and the system is lossless. The heat absorbed at T_1 is equal to the work done by the cold expander:

$$W_e = Q_1. \tag{5}$$

Then by conservation of energy the heat rejected at T_0 is equal to the work of compression:

$$W_p = Q_0. \tag{6}$$

If the work of expansion is recovered, as it is in an ideal system, the net work input is $W_n \equiv W_p - W_e$ or by substituting in (5) and (6):

$$W_n = Q_0 - Q_1. \tag{7}$$

Again in a lossless system, entropy is conserved:

$$Q_1/T_1 = Q_0/T_0. \tag{8}$$

Combining (7) and (8) yield the same efficiency and P for an ideal Stirling cycle as for a Carnot cycle:

$$\square_s \equiv Q_1/W_n = (T_0/T_1 - 1)^{-1} \tag{9}$$

and
$$P_s = 1/\square_s = T_0/T_1 - 1. \tag{10}$$

In an idealized OPT the cold mechanical piston is replaced by a compressible gas volume within the pulse tube. This volume of gas acts as a displacer, adiabatically transferring work from the cold end to the hot end of the pulse tube. (Compressing the gas displacer takes some work; but, this is recovered when the displacer re-expands later in the cycle.) The mechanical work of expansion transferred to the hot end is not recovered but is dissipated as heat at the hot end of the pulse tube. The orifice and reservoir provide the phase shift required to produce the work flow. Other than what happens to the expansion work, the ideal OPT behaves just as a Stirling cycle. Thus

$$W_p = Q_0 \tag{11}$$

and
$$W_e = Q_1. \tag{12}$$

But W_e is dissipated rather than recovered:

$$Q_e = W_e. \tag{13}$$

So the net input work becomes

$$W_n = W_p. \tag{14}$$

Since we are considering an ideal system, it is still lossless except for Q_e. Thus entropy is still conserved at both ends of the regenerator:

$$Q_1/T_1 = Q_0/T_0. \tag{15}$$

Using (11), (14), and (15) results in an efficiency and P of

$$\square_t \equiv Q_1/W_n = T_1/T_0 \tag{16}$$

and
$$P_t = 1/\square_t = T_0/T_1. \tag{17}$$

Comparing (17) to (4) yields:

$$P_t = P_c + 1. \tag{18}$$

Thus a OPT can never achieve the efficiency of a Carnot cycle. This is the direct result of the expansion work not being recovered. However it is interesting to note that as the temperature ratio (T_0/T_1) becomes bigger, the efficiency of the OPT approaches Carnot efficiency.

Because the OPT requires work to be dissipated, the OPT operates on an inherently irreversible cycle. Irreversible cycles can never achieve Carnot efficiency by the Second law of thermodynamics. Thus (18) is just a statement of this irreversibility.

This discussion holds only for ideal refrigerators. Typically practical refrigerators have specific powers that are x10 or greater than ideal. It is difficult to extrapolate from the conclusion reached here to practical refrigerators where various other losses overwhelm the inherent loss of an ideal orifice pulse tube refrigerator. Except for very small temperature ratio machines, there is nothing discussed here that prevents a practical OPT refrigerator from being as efficient or more efficient than a practical Stirling refrigerator.

References:

1. Radebaugh, R. A Review of Pulse Tube Refrigeration *Adv Cryo Eng*, (1990) 35B 1192-1205.

2. Kasuya, M., Nakatsu, M., Geng, Q., Yuyama, J., and Gato, E. Work and Heat Flows in a Pulse-Tube Refrigerator *Cryogenics* (1991) 31 786-790.

Figure Captions:

Figure 1.
Schematic representation of the energy and work flow in a Carnot cycle where Q_1 is the heat absorbed at the cold temperature, T_1; Q_0 is the heat rejected at the hot temperature, T_0; and W_n is the net work input.

Figure 2.
Schematic representation of the □-Stirling and Orifice Pulse Tube refrigerators where Q_0, Q_1, T_0, T_1 are as defined in figure 1 and W_p is the work done by the compressor, W_e is the expansion work recovered and Q_e is the expansion work dissipated.